Synopsis of the thesis entitled

OPTIMAL ORDERING INVENTORY POLICIES FORDYNAMIC PURCHASE PRICES

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1. Introduction

Inventory is a stock of materials that is being held to satisfy present and future needs of any organization. In order to meet the demand of the customers, every business must keep inventory of goods. Maintaining inventory is also an important component of supply chain. The three types of inventory held in manufacturing company are raw materials inventory, work-in-progress inventory and finished goods inventory. An under-stocking of goods in any business results in lost sale which creates dissatisfied customers and in some cases shortages that occurred will be met afterwards. On the other hand, over-stocking increases the inventory holding costs, which include cost of money tied up, cost of storage etc. Hence in every inventory problem there are two questions which need to be answered. They are, (i) How much to order and (ii) when to order? These two questions should be answered in such a way that the total cost (profit) of the inventory is minimum (maximum). Depending on the situation, complexity of the inventory problem varies significantly. Some of the complex inventory problems require sophisticated mathematical techniques and advanced computer algorithms to get appropriate solutions.

W. Harris [9] developed the first mathematical inventory model, which is based on certain assumptions. This famous model was extended by many authors by relaxing some of the assumptions. One of the key assumptions of the model developed by Harris was constant purchase price. For better representation of real word situations, one must relax the static purchase price assumption.

There are many real-life examples which violate the static price assumption. One among them is the discount price offer by the supplier to the retailer. This leads to two price inventory problem. One of the reasons to offer the discount is to decrease the level of excess inventory and to increase the cash flow. Under such discount pricing strategy of the supplier, the retailer must change the regular ordering policy to get the benefit of reduced purchase price. Another example of two price problem that exists in inventory literature is single announced price change. Apart from this, price of the commodity may increase or decrease at a constant rate or continuously during the planning horizon. Determination of an optimal policy without considering the price change gives considerably different result than that of varying price. Hence developing an optimal

inventory policy for the various situations of price fluctuation is necessary.

The objectives of this thesis are:

(i) To investigate applications where the purchase price variations occur.

(ii) To develop optimal inventory policies to various types of price fluctuation through different methods which are not available in the literature.

(iii) Compare the developed optimal inventory policy with EOQ approach and other models.

(iv) To carry out certain sensitivity analysis.

2. Literature Review

The price change inventory problem was first studied by Naddor [20], who proposed the infinite horizon EOQ model with the assumption that price change occurs at the end of an EOQ cycle. Lev and Soyster [17] developed a finite horizon inventory model and determined the optimal ordering policies when information about an ensuing price rise was known. An improved algorithm for determining the optimal policy for the model of Lev and Soyster [17] was provided by Goyal [8]. Taylor and Bradley [24] extended Naddor's [20] model and found optimal ordering strategies for situations where the price increase did not coincide with the end of the EOQ cycle. Lev and Wiess [18] also developed a comprehensive model that covered both finite and infinite horizons under purchase price increase and decrease. Their algorithm for solving the finite horizon EOQ model with price change was modified by Andre Gascon [7]. Huang and Kulkarni [10] have discussed an infinite horizon deterministic inventory model under an announced price increase. EOQ model with a known price increase and partial back-ordering was developed by Taleizadeh and Pentico [23].

In contrast to single purchase price change models many researchers have worked on inventory models with continuous price change. The optimal ordering policy when purchase price increases due to inflation have been studied by Buzacott [4] and Erel [5]. Buzacott [4] considered an infinite horizon model and assumed compound increasing price and setup cost, whereas in Erel's [5] model a compound increasing price during a finite planning horizon is considered. Khouja and Park [16] developed a closed form approximate solution for an inventory model with continuous decrease in purchase price. They made a restrictive assumption of the equal cycle time. Their model was improved

by Khouja and Goyal [15] by considering cycle times of varying lengths. Teng and Yang [25] extended the classical EOQ model to allow for time-varying demand and unit purchase cost. They considered shortages and partial backlogging in their model. Panda [21] considered an optimal inventory policy for the scenario of price sensitive demand in which both demand and unit cost of the item is assumed to be decreasing with time. Junfang Y et al. [13] developed an efficient algorithm for the inventory system for high-tech companies where unit price of the items decreases continuously. Kamali H.R. et al. [14] presented deterministic and Metaheuristic Solutions for closed-loop supply chains with continuous price decrease.

These deterministic models have been extended to incorporate the situation where the duration of the change in price is a random variable (See, Arcelus et al., [2,3]). The change in price is also a repetitive process that can prevail for a long duration. The structure of price uncertainty over an infinite horizon under risk-sharing agreements among supply-chain partners was developed by Arcelus et al. [1] as a general model by incorporating the probability structure of the duration of change through two random variables.

Many researchers developed optimal inventory policies for the scenario of repetitive special sales. Friend [6] developed an optimal policy for a continuous review inventory system with randomly arriving discount prices. The model developed by Friend was extended by Hurter and Kaminsky [11]. Hurter and Kaminsky [12] further discussed a model in which price discounts are offered according to a Poisson process. Later, Moinzadeh [19] analyzes the model of Hurter and Kaminsky [12], he considered demand rate as a constant. Tajbakhsh et al. [22] considered a continuous review inventory model in which they assumed that discount offerings follow a Poisson process and discount price is a discrete random variable with a known distribution.

3. Summary of the Work Done

The work carried out in this thesis considers impact of variations in purchase prices. Two price problems, three price problems, multiple prices problems and continuous price problems are considered in the thesis. Price may change according to random time or at

the time of the order. The work of the thesis is distributed in seven chapters. In Chapter 1 introduction to inventory and basic terminology are considered. In this chapter significance of the field of research is elaborated and area of applications are mentioned. The literature review related to the concepts discussed in the thesis is also considered in this chapter.

Chapter 2 discusses an optimal inventory policy for the situation of varying purchase price due to the fluctuation of exchange rate. An attempt has been made to model the influence of the risk-sharing agreements for the retailers' inventory problem. The exchange rates are divided into certain classes. According to the range of possible exchange rates, a fixed number of intervals are considered to fix a specific purchase price in the agreement. That is, the entire range of possible values of exchange rates is divided into mutually exclusive and exhaustive classes. For each class, a purchase price is agreed upon between two parties. The stochastic variations in purchase prices, due to the fluctuation of exchange rate, are modelled through a Markov chain. The resulting purchase and inventory problem are analyzed by identifying a regenerative cycle. The variables of the model are modified to suit the discrete nature of the business working system. Thus, straightforward optimization techniques are not applicable. In order to apply regenerative cycle; a special-order cycle is defined. The various ways in which these special-order cycles complete is identified along with their probabilities. Hence expected profit of the special-order cycle is determined. A suitable algorithm is devised to solve the dynamic problem. An optimal selling price that maximizes the expected profit per unit time is also discussed. Further, optimal ordering policies under no stock-out conditions are derived with an optimal uniform demand corresponding to the optimal selling price. Through sensitivity analyses, differences in profit function with respect to carrying cost fraction, setup costs, and purchase prices are shown. An investigation into the loss that could occur if this model solution is not implemented is also made through numerical illustrations. A discussion of a special case of two-purchase price scenario gives additional insight into the problem. **An article based on this chapter is published in OPSEARCH journal**.

The Chapter 3 considers an optimal inventory policy for the continuously changing purchase price due to fluctuation of the exchange rate. Here no risk sharing agreement is assumed. The purchase price is directly depending on the exchange rate. Further the

differences with the model discussed in Chapter 2 are (i) planning horizon is finite, (ii) time variable is continuous, (iii) purchase prices not to vary over limited fixed points, and (iv) purchase price process is modelled by geometric Brownian motion process and Gaussian process. The real exchange rate data of INR vs USD is used to estimate the model parameters. The optimal number of order and optimal order quantities were obtained by minimizing the total cost over the finite planning horizon. Initially the Brownian motion process model parameters are estimated by method of the maximum likelihood method using data on real exchange rates. The impact of continuously changing purchase price because of fluctuating exchange rate is identified by comparing the optimal expected cost for constant and fluctuating purchase price. Brownian process is also examined to model the exchange rate and model solutions are compared with the solutions obtained by geometric Brownian motion process. The difference in optimal expected cost between the model with constant purchase price and our model is found for different parametric values through sensitivity analysis. Through the numerical work, it is found that for larger demands, the optimal expected cost of the two models differs largely.

In Chapter 4, an ordering and pricing policies of a profit-maximize retailer is developed, when supplier offers discount on the regular price for a long period, which are repeated and last for a random interval of time. An optimal ordering policy for two purchase price problem is considered. Time is taken as discrete according to the business transactions and other aspects into considerations like accounting and administrative constraints. The stochastic variation in purchase price is modelled through a Markov chain. Discrete nature of time variable along with the application of discrete Markov chain isolates our work from the earlier literatures to solve the temporary price problem. The resulting purchase process and inventory problem are analyzed by identifying a regenerative cycle. Each of the regenerative cycles may complete in one of the six different possibilities with corresponding probabilities. By taking respective cost functions of these possibilities the expected cost of a regenerative cycle is determined. It is also proved that the number of regular orders in each cycle follows a geometric random variable. Various theorems are proved to assist the desired result. Optimal ordering policies are developed by maximizing the long run expected profit per unit time. Numerical examples are considered to illustrate the theoretical results. Sensitivity analysis was carried out to find the percentage variations in expected profit per unit time

due to the variations in model parameters. It was found that discount parameter has more effect on the optimal expected profit per unit time.

Chapter 5 considers an optimal inventory policy of the retailer in response to first order discount offer by the supplier. Unlike many of the two price problems, possibility of only one order to be placed under discount or reduced price is considered. This is also applicable when retailer has last chance to place an order under discount price or regular price. After placing order price increase occurs, hence all following orders are in increased price. Both the situations mentioned are represented by this model. Yearly demand of the customer is considered as constant and known to the retailer. A closed form solution to the optimal number of orders and optimal order quantities is developed. The optimal number of orders after the first purchase is derived and stated in a theorem. The relation between first order quantity and discount price of the supplier is derived. Numerical example and sensitivity analysis are considered to highlight the important characteristic of the developed model.**An article based on this chapter is published in International Journal of Engineering and Research Technology journal**.

This chapter also discusses another situation where the first order is regular, and all following orders are discounted. This scenario exits in many business situations. In some business transactions for every order a coupon is given and that can be used in the next order. A finite horizon model with constant yearly demand is considered. Convexity of the cost function is established. The closed form solution for optimal order quantities and an optimal number of orders is obtained by minimizing the total cost. An algorithm is given to arrive at the optimal number of orders and optimal order quantities and hence optimal cost. Numerical analysis is carried out to support the theoretical results.

An optimal inventory policy when supplier offers subsequent discount to the retailer is considered in Chapter 6. A special pricing strategy of the wholesaler which results in subsequent smaller amount of discount is considered in this chapter. A finite horizon model with constant yearly demand is considered. Two models are considered here. The first one considers three-price problem, that is, after the first order, a discount is given for the second order, and further discount is given for all subsequent orders, starting with the third order. The second model is exponential discount or price increase where the purchase price decreases exponentially after each order or the price increases

exponentially after every order. Hence price for every order will be different. A finite horizon model with constant yearly demand is considered. In case of the first model, the closed form solutions for optimal order quantities for given number of orders in the planning horizon is obtained by minimizing the total cost. An algorithm is given to arrive at the optimal number of orders and optimal order quantities and hence optimal cost. Numerical analysis is carried out to support the theoretical results. A significant percentage difference in cost between the developed optimal policy and the EOQ policy highlights the importance of the developed optimal policy. It is found that a large percentage difference exists for a smaller planning horizon. In the second model, a subsequent change in the purchase price is considered in an exponential format. Here we consider monotonic price change problem. That is, one scenario is that the purchase price is gradually discounted on every order and in the other scenario purchase price is gradually increasing on every order. The multiple decision variables are to be determined so as to minimize the total cost. The resulting objective function is in a quadratic form. We have derived the closed form solutions for the optimal order quantities when number of orders in the planning horizon is given. Certain properties of these solutions are shown. An expression for the optimal cost function is derived. Further an algorithm is given to determine the optimal number of orders in the planning horizon, which completes the process of determining the optimal policy for the model. A numerical illustration is also given.

The thesis concludes with the Chapter 7, where conclusion and scope for future research is considered.Important findings are also included as a part of Chapter 7, which include:

1. While modelling the purchase price variation through discrete time Markov chain by considering price elastic demand function, it was observed that the effect of price elasticity is more on profit than other parameters. It is also found that there was a significant loss when optimal policies and optimal prices discussed in our work were not used. A solution is given to the inventory problem under risk sharing agreement.
2. For a discount price problem, it was found that discount parameter has more effect on the optimal expected profit per unit time.
3. In Chapter 3, it is found that for larger demands, the difference in the optimal expected cost for constant and varying purchase is more. A comparison of the model solutions obtained by modelling the exchange rate as a geometric Brownian motion

process is made with solutions obtained using Gaussian Process. Coefficient of variations of the total cost for both the methods are compared and found that GBM performs better than Gaussian process.

4. For subsequent price discount problem, a significant difference was found between developed optimal policy and EOQ policy. We also observed that this significant difference decreases as planning horizon increases. Similar observation was also observed for the first purchase discount problem.
5. For exponentially decreasing/increasing problem (depending on number of orders) an increase/decrease in order quantities are observed with decrease/increase in purchase price.

Finally scope for further research is mentioned in the last chapter. Various extension of the work carried out in the thesis is mentioned here. New dimension of the work is also mentioned.

List of Published Papers

1. Kharvi, S., Pakkala, T.P.M., Srinivasan, G.: Ordering policies under currency risk sharing agreements: a Markov chain approach. OPSEARCH. 56(3), 945-964 (2019)
2. Kharvi, S., Pakkala, T.P.M.: An Inventory Policy for Discount only for the First Purchase. International Journal of Engineering Research and Technology. 9(6), 149-153 (2020)

List of Presented Papers

1. Presented a paper **"Production lot sizing problem using Markov decision process"** in National seminar on Statistical inference and Stochastic modelling, Karnataka University, Dharwad.
2. Presented a paper **"Inventory Policies under Stochastic Purchasing Price"** in National Conference on Science and Technology", Mangalore University.

List of Communicated Papers

1. Kharvi, S., Pakkala, T.P.M.: An Optimal Inventory Policy when Purchase Price Follows Geometric Brownian Motion Process. OPSEARCH.
2. Kharvi, S., Pakkala, T.P.M.: An Optimal Inventory Policy for Subsequent Price Reduction Problem. The International Journal of Operations and Quantitative Management.
3. Kharvi, S., Pakkala, T.P.M.: Inventory Policies under Stochastic Purchase Price. RACSAM.

References:

1. Arcelus, F.J., Pakkala, T.P.M., Srinivasan, G.: A purchasing framework for B2B pricing decisions and risk-sharing in supply chains. Decision sciences. 33 (4), 645–672 (2002)
2. Arcelus, F.J., Pakkala, T.P.M., Srinivasan, G.: A retailer's decision process when anticipating a vendor's temporary discount offer. Computers & Industrial Engineering. **57**, 253–260 (2009).
3. Arcelus, F.J., Pakkala, T.P.M., Srinivasan, G.: On the interaction between retailer's inventory policies and manufacturer trade deals in response to supply-uncertainty occurrences. Annals of Operations Research. **143,** 45–58 (2006).
4. Buzacott, J. A.: Economic Order Quantities with Inflation. Operational Research Quarterly.**26**(3), 553-558 (1975)
5. Erel, E.: The effect of continuous price change in the EOQ. Omega, 21(5), 593-594 (1992).
6. Friend, J. K.: Stock control with random opportunities for replenishment. Operational Research Quarterly. 11(3), 130–136 (1960)
7. Gascon, A.: On the Finite Horizon EOQ Model with cost changes. Operations research, **43**(4), 716-717 (1995)
8. Goyal, S. K.: A Note on the Paper: An Inventory Model with Finite Horizon and Price Changes. Journal of the Operational Research Society,**30**(9), 839-840 (1980)
9. Harris, F. W.: How many parts to make at once. The Magazine of Management. 10(2), 135-152, 1913.
10. Huang, W., Kulkarni, V. G.: Optimal EOQ for announced price increases in infinite horizon. Operations Research, 51(2), 336–339 (2003).

11. Hurter, A. P., Kaminsky, F.C.: Inventory control with a randomly available discount purchase price. Journal of the Operational Research Society. 19 (4), 433-444 (1968 a)
12. Hurter, A. P., Kaminsky, F.C.: Inventory control with random and regular replenishment. Journal of Industrial Engineering. 19(8), 380-385 (1968 b)
13. Junfang, Y., Deniz, M., Bhaba, R.S.: An integrated multi-stage supply chain inventory model under an infinite planning horizon and continuous price decrease. Computers & Industrial Engineering, 61, 118- 130 (2011).
14. Kamali, H.R., Sadegheih, A., Vahdat-Zad, M. A., Khademi-Zare, H.: Deterministic and Metaheuristic Solutions for Closed-loop Supply Chains with Continuous Price Decrease. International Journal of Engineering, 27(12), 1897-1906 (2014)
15. Khouja, M., Goyal, S. K.: Single item optimal lot sizing under continuous unit cost decrease. International Journal of Production Economics. 102, 87-94 (2006)
16. Khouja, M., Park, S.: Optimal lot sizing under continuous price decrease. Omega. 31, 539-545 (2003)
17. Lev, B., Soyster, A. L.: An Inventory Model with Finite Horizon and Price Changes. Journal of the Operational Research Society. **30**(1), 43-53 (1979)
18. Lev, B., Weiss, H.J.: Inventory Models with Cost Changes. Operations Research, **38**(1), 53-63 (1990)
19. Moinzadeh, K.: Replenishment and stocking policies for inventory systems with random deal offerings. Management Science. 43 (3), 334-342 (1997)
20. Naddor, E.: Inventory systems. Wiley, New York (1966)
21. Panda, S: Optimal pricing replenishment policy in a declining price sensitive environment under continuous unit cost decrease. International Journal of Mathematics in Operations Research. 3 (4), 431–450 (2011)
22. Tajbakhsh, M. M., Lee, C.G., Zolfaghari, S.: An inventory model with random discount offerings. Omega. 39 (6), 710–718 (2011)
23. Taleizadeh, A.A., Pentico, D.W.: An economic order quantity model with a known price increase and partial backordering. **228**(3), 516-525 (2013)
24. Taylor, S.G., Bradley, C.E.: Optimal ordering strategies for announced price increases. Operations Research. **33**(2), 312–325 (1985)

25. Teng, J.T., Yang, H.L.: Deterministic economic order quantity models with partial backlogging when demand and cost are fluctuating with time. Journal of the Operational Research Society. 55, 495-503 (2004)

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